

IBDP PHYSICS Internal Assessment

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“To what extent does the length of the string affect the period of a simple pendulum?”



INTRODUCTION

The original aim for this investigation was to “investigate the simple pendulum”. There are many variables one could look into, such as displacement, angle, damping, mass of the bob etc. The most interesting variable, however, is the length of the swinging pendulum. The relationship between the length and the time for one swing (the period) has been researched for many centuries, and has allowed famous physicists like Isaac Newton and Galileo Galilei to obtain an accurate value for the gravitational acceleration ‘g’. In this report, we will replicate their experiment, and we will try to find an accurate value for ‘g’ here in Pisa. We will then compare this value with the commonly accepted value of 9.806 m/s^2 [NIST, 2009]

A CLOSER LOOK AT OUR VARIABLES

In this investigation, we varied the length of the pendulum (our independent variable) to observe a change in the period (our dependent variable). In order to reduce possible random errors in the time measurements, we repeated the measurement of the period three times for each of the ten lengths. We also measured the time for ten successive swings to further reduce the errors. The length of our original pendulum was set at 100 cm and for each of the following measurements, we reduced the length by 10 cm.

THEORY

A simple pendulum performs simple harmonic motion, i.e. its periodic motion is defined by an acceleration that is proportional to its displacement and directed towards the centre of motion. It can be shown that the period T of the swinging pendulum is proportional to the square root of the length l of the pendulum:

$$T = 4\pi^2 \sqrt{\frac{l}{g}} \quad (\text{Hyperphysics, 2009}), \quad [1]$$

with T the period in seconds, l the length in metres and g the gravitational acceleration in m/s^2 . Our raw data should give us a square-root relationship between the period and the length. Furthermore, to find an accurate value for ‘g’, we will also graph T^2 versus the length of the pendulum. This way, we will be able to obtain a straight-line graph, with a gradient equal to $4\pi^2 g^{-1}$.

EQUIPMENT AND METHOD

For this investigation, we had access to limited resources; clamps, stands, a metre ruler, a stopwatch, a metal ball (a.k.a. bob), and some string. The experimental set-up was equal to the diagram, shown in figure 1 (Practical Physics, 2009).

As stated earlier in the introduction, it was decided to measure the time for ten complete swings, in order to reduce the random errors. These measurements would be repeated two more times, and in total ten successive lengths were used, starting from one metre, and decreasing by 10 cm for each following measurement.

A metre ruler was used to determine the length of the string. One added difficulty in determining the length of the pendulum was the relative big uncertainty in finding the exact length, since the metal bob added less than a centimeter to our string length, measured from the bob's centre. This resulted in an uncertainty in length that was higher than one would normally expect.

The table clamp was used to secure the position of the tripod stand, while the pendulum was swinging.

After the required measurements, one experiment was carried out to find the degree of damping in our set-up. Damping always occurs when there is friction, but exactly how significant the degree of damping in our experimental set-up was, remained uncertain. Depending on the degree of damping, it may or may not have a significant effect on our measurements.

All measurements were taken under the same conditions, using the same metal bob, the same ruler, in the same room, and at approximately 26 degrees Celsius.

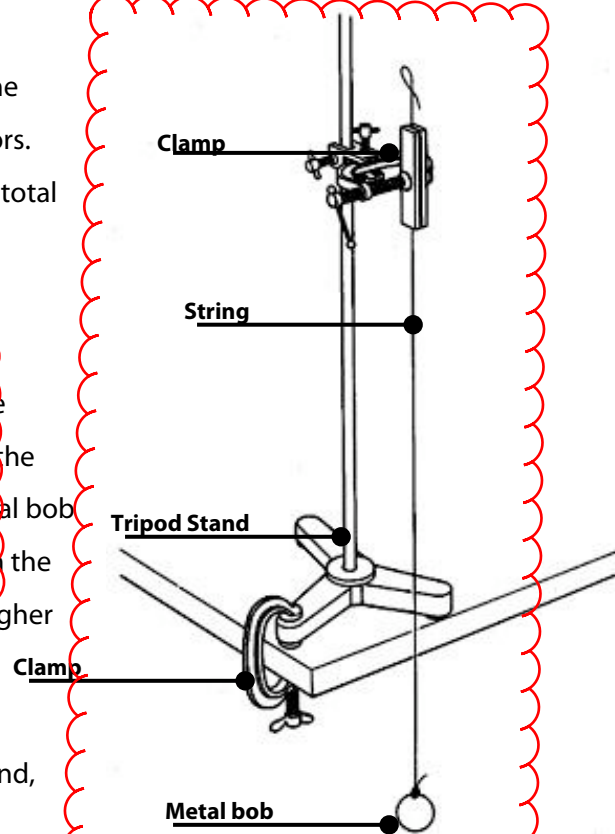


Figure 1: Diagram of the set-up for this experiment

DATA COLLECTION

Time taken for 10 swings (in seconds) ± 0.4 seconds ¹			Length (in metres) ± 0.005 metres ²
First trial	Second Trial	Third Trial	All Trials
7.5	7.7	7.3	0.10
9.7	9.9	9.5	0.20
11.4	11.6	11.2	0.30
13.3	13.1	13.5	0.40
14.5	14.9	14.1	0.50
16.0	16.6	15.4	0.60
17.1	17.0	17.2	0.70
18.0	18.3	17.7	0.80
19.0	19.1	19.0	0.90
20.2	20.0	20.4	1.00

These raw data need to be processed, before we can continue our analysis. First of all, we need to find the average of the three trials, which will reduce our error, stated in the header. Secondly, we will have to find the time for one swing (or one period), which will reduce our absolute error, but not our percentage error.

It should also be noted, that for all our measurements, we used a constant, and small, angle of maximum displacement (a.k.a. amplitude). The angle was kept between 5° to 7° , small enough to ignore the friction present in our experimental set-up.

Apart from these measurements, we also did one more experiment, which was to see how much damping was present in our set-up. It took, on average, between 100 and 150 swings, before the motion had seemed to stop. This showed that there was damping present, but this did not significantly affect our measurement of just ten swings.

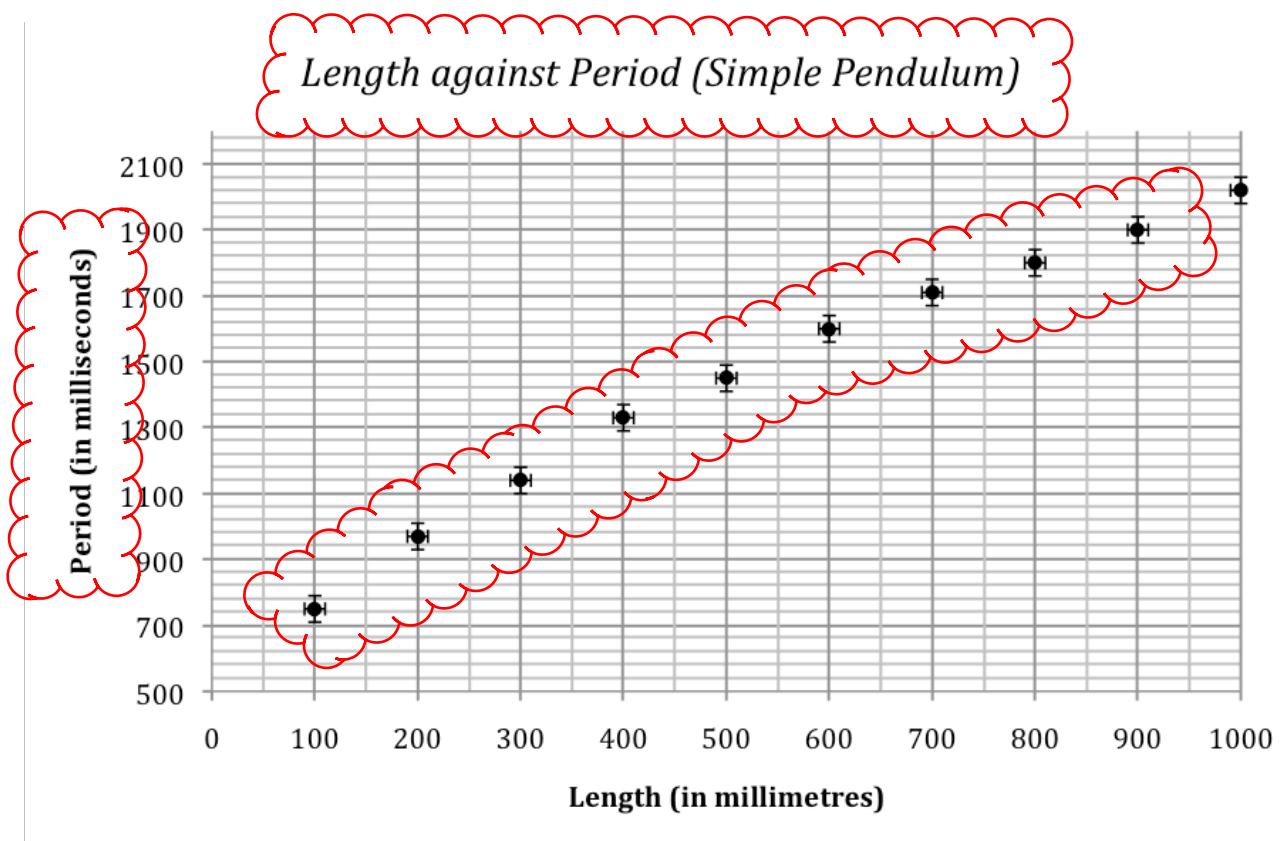
The table on the following page shows the processed data and the uncertainties.

¹ The uncertainty in time was obtained by comparing the spread for the different measurements. The time measurement for the 0.50 metre length, had the largest spread (± 0.4 seconds), and was therefore used as the uncertainty in the time measurement.

² The theoretical uncertainty in the length measurement would be 0.05 cm (we used a metre ruler). However, in the experimental set-up, the two end points (the one tied to the clamp, and the one tied to the metal bob) gave rise to a bigger uncertainty, as the exact end-points could not be precisely determined. We estimated the uncertainty in length to be 0.5 cm, or 0.005 metres.

Average time taken for one swing (in seconds) ± 0.04 seconds	Percentage Uncertainty In Period (in seconds)	Length (in metres) ± 0.005 metres
0.75	5.3%	0.10
0.97	4.1%	0.20
1.14	3.5%	0.30
1.33	3.0%	0.40
1.45	2.8%	0.50
1.60	2.5%	0.60
1.71	2.3%	0.70
1.80	2.2%	0.80
1.90	2.1%	0.90
2.02	2.0%	1.00

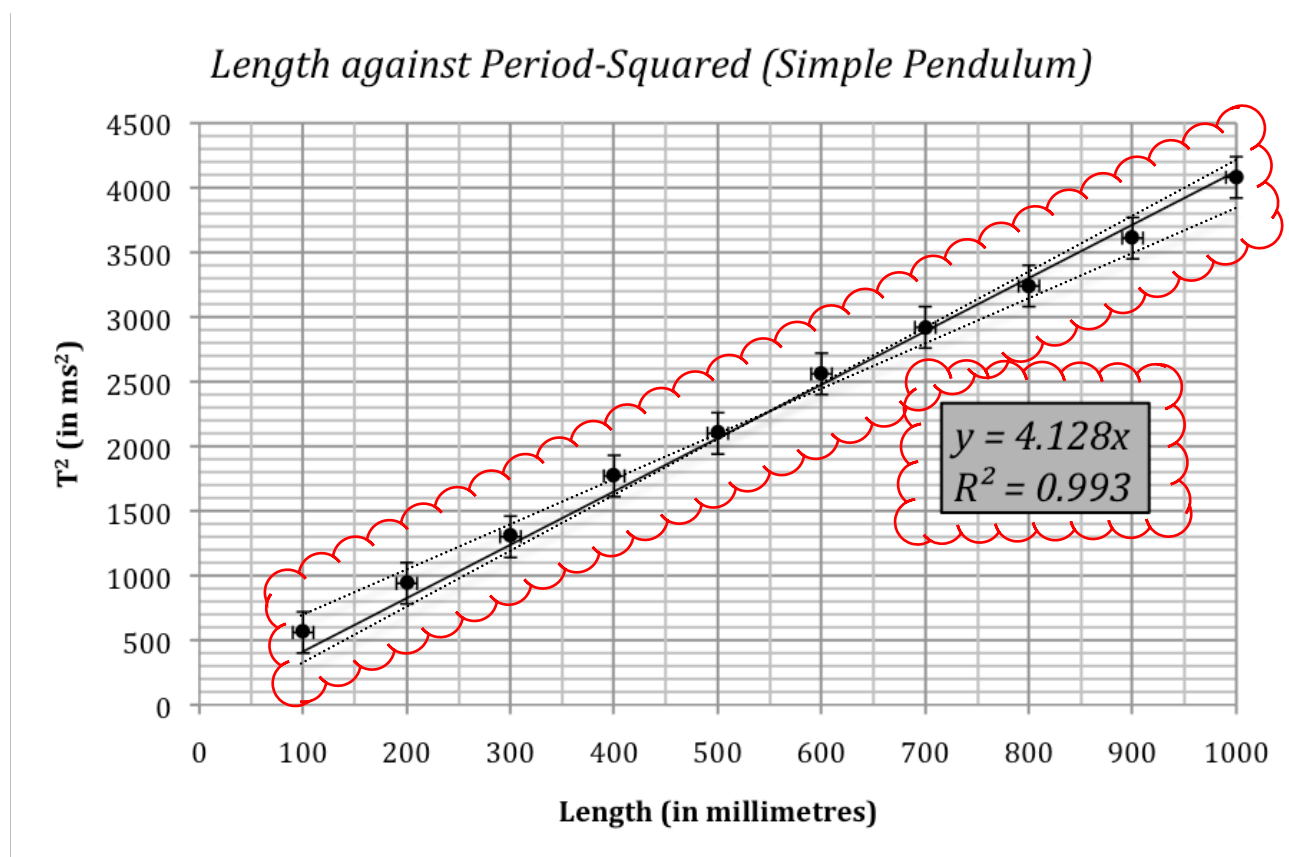
When graphing the data below*, the relationship between the variables used is clearly not a linear one. The suggested square-root relationship is very obvious, and to linearise this curve, we will have to interchange and modify our axes.



* Graphing the period T in milliseconds and the Length in millimeters was a requirement of the software used (Microsoft Excel), as the minimum error bar value is one.

T² (Period Squared) (in seconds²)	% Uncertainty³ in T²	Absolute Uncertainty in T² (in seconds²)	Length (in metres) ±0.005 metres
0.56	10.6%	0.06	0.10
0.94	8.2%	0.08	0.20
1.30	7.0%	0.09	0.30
1.77	6.0%	0.11	0.40
2.10	5.6%	0.12	0.50
2.56	5.0%	0.13	0.60
2.92	4.6%	0.13	0.70
3.24	4.4%	0.14	0.80
3.61	4.2%	0.15	0.90
4.08	4.0%	0.16	1.00

Based on the theory of Simple Harmonic Motion and equation [1] presented earlier, we expect a linear relationship between T² and Length. When graphing these two modified variables, we expect the regression line to be linear, passing through point (0,0) and with a gradient equal, or close to $4\pi^2g^{-1}$.



CONCLUSION AND EVALUATION

Graphing the length against T^2 clearly shows a linear relationship, in agreement with the theory. The actual line of best fit does not go through (0,0) which suggests a systematic error in our experiment. But when graphing a line of best fit, with the condition it should pass through (0,0), we find a line with a gradient of 4.128 and a correlation coefficient of 0.993, which further suggests a very strong linear correlation between our chosen variables.

Our value for 'g' can be calculated by dividing $4\pi^2$ with the gradient of the line of best fit;

$$g_{\text{best}} = 9.56 \text{ m/s}^2 \pm 0.6$$

The uncertainty in this value was found, by taking half the difference of the lowest possible value for 'g' and the highest possible value for 'g':

$$g_{\text{min}} = \frac{4\pi^2}{\text{gradient}_{\text{max}}} = \frac{4\pi^2}{\frac{(4.24-0.50)}{(1.0-0.1)}} = 9.50 \text{ m/s}^2$$

$$g_{\text{max}} = \frac{4\pi^2}{\text{gradient}_{\text{min}}} = \frac{4\pi^2}{\frac{(3.92-0.62)}{(1.0-0.1)}} = 10.77 \text{ m/s}^2$$

Comparing our calculated value for the gravitational acceleration 'g' with the accepted theoretical value gives us an error of 2.5%, well within the error margins that we calculated. This is a reasonable result, given the equipment and the time constraints that we faced.

Looking at our graph, we cannot identify any outliers. However, our data values suggest a line of best fit that does not pass through (0,0). When we do fit a linear regression onto our data values, that passes (0,0), we see that the line does not 'hit' all the horizontal error bars (the uncertainty in the length). This may suggest a systematic error in the measurement of the length of our pendulum.

Furthermore, this experiment had to be carried out in about one hour, with very basic equipment. This, perhaps, contributed to the slight difference in the value for 'g' that we found.

FUTURE IMPROVEMENTS

To reduce the systematic error in the length measurements, one should take accurate measurements of the diameter of the metal bob used. In our experiment, it looks as if we systematically used a length for the pendulum that was too short. If we add 1 cm to our data, we would get a value for 'g' that is equal to

³ The percentage uncertainty in T^2 is twice the percentage uncertainty in T

the theoretical value of 9.806 m/s^2 . There could be another explanation, which is the fact that our experiment was carried out in Pisa. The theoretical value we used, is the average value for 'g' on Earth, and may be slightly different from the one we are measuring, since Pisa has latitude: 43.681° and longitude: 10.39° (Flightpedia, 2009).

Instead of three measurements, we could have taken five measurements, as it would not take too much extra time, and this would further reduce our uncertainty in the measurement of the period of swing. Alternatively, we could have measured the time for 20 swings, instead of 10 swings, which would also reduce our uncertainty in time.

Lastly, we could use a photogate in the future, to measure the period with higher precision. A nice extension to this experiment would be the use of different metal bobs, of different diameter and/or mass. This would allow us to calculate the effect of air resistance on this experiment.

REFERENCES

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